• Dispersion causes the duration and shape of an optical pulse to change in the course of propagation, causing bit errors in reception.

• Dispersion is typically measured as a time spread per distance traveled (s/km)

Intersymbol interference

For no overlapping of light pulses down on an optical fiber link, the digital bit rate $B_T$ must be less than the reciprocal of the broadened (through dispersion) pulse duration ($2\tau$). Hence:

$$B_T \leq \frac{1}{2\tau}$$

Another more accurate estimate of the maximum bit rate for an optical channel with dispersion may be obtained by considering the light pulses at the output to have a Gaussian shape with an RMS width of $\sigma$.

The maximum bit rate is given approximately by
The conversion of bit rate to bandwidth in hertz depends on the digital coding format used.

\[ B_{\tau(\text{max})} \approx \frac{0.2}{\sigma} \text{ bit.s}^{-1} \]

For nonreturn to zero code

\[ B_{\tau(\text{max})} = 2B \]

For return to zero code

The amount of pulse broadening is dependent upon the distance the pulse travel within the fiber. In absence of mode coupling, the pulse broadening increases linearly with fiber length and thus the bandwidth is inversely proportional to distance.

A measure of the information capacity of an optical waveguide is usually specified by **bandwidth-distance product** in MHz.km.

<table>
<thead>
<tr>
<th>Fiber type</th>
<th>Bandwidth-distance product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step index multimode</td>
<td>20 MHz.km</td>
</tr>
<tr>
<td>Graded index</td>
<td>20 GHz.km</td>
</tr>
<tr>
<td>Single mode</td>
<td>100 GHz.km</td>
</tr>
</tbody>
</table>
1- Intramodal (Chromatic) dispersion

There are two types of intramodal dispersion. The first type is material dispersion. The second type is waveguide dispersion.

1-1 Material dispersion occurs because the spreading of a light pulse is dependent on the wavelengths’ interaction with the refractive index of the fiber core. Different wavelengths travel at different speeds in the fiber material. Different wavelengths of a light pulse that enter a fiber at one time exit the fiber at different times. Material dispersion is a function of the source spectral width. The spectral width specifies the range of wavelengths that can propagate in the fiber. Material dispersion is less at longer wavelengths.

- It is said to have material dispersion when the second derivative of refractive index with respect to wavelength is not zero \( \left[ \frac{d^2n_1}{d\lambda^2} \neq 0 \right] \).
CD causes the shorter $\lambda$ to travel faster than the longer $\lambda$.

**What is Group Velocity?**

- Group Velocity $(v_g)$ is considered as the velocity of energy propagating in the direction of the axis of the guide fiber.
- In order to convey intelligence; Modulation is done. When done, there are group velocities those must be propagating along the fiber.
- The waves of different frequencies in the group will be transmitted with slightly different velocities. $v_g = \frac{d\omega}{d\beta}$.

- The group delay $\tau_g$ is given by:

\[ \tau_g = \frac{d\beta}{d\omega} = \frac{1}{C} \left( n_1 - \lambda \frac{dn_1}{d\lambda} \right) \]  

- The pulse delay $\tau_m$ due to material dispersion in a fiber of length $L$ is therefore:

\[ \tau_m = \frac{L}{C} \left( n_1 - \lambda \frac{dn_1}{d\lambda} \right) \]

For a source with RMS spectral width $\sigma_\lambda$ and a mean wavelength, the RMS pulse broadening $\sigma_m$ due to material dispersion [which may be obtained from the expansion of Taylor series about $\lambda$ as]

\[
\sigma_m = \sigma_\lambda \frac{d\tau_m}{d\lambda} + \sigma_\lambda \frac{d^2\tau_m}{d\lambda^2} + \ldots
\]
Hence the pulse broadening may be evaluated by considering the dependence of $\tau_m$ on $\lambda$ where from eq (2):

$$\frac{d\tau_m}{d\lambda} = \frac{-L\lambda}{c} \frac{d^2 n_1}{d\lambda^2}$$

And approximately

$$\sigma_m \cong \sigma_\lambda \frac{d\tau_m}{d\lambda} = \frac{\sigma_\lambda}{c} L \frac{\lambda}{L} \frac{d^2 n_1}{d\lambda^2} = \sigma_\lambda L M$$

where:

- Where $M$ is the material dispersion parameter in (ps.nm$^{-1}$.km$^{-1}$). It may be observed that the material dispersion tends to zero in the longer wavelength region around 1.3 $\mu$m (for pure silica).
- It can be reduced either by choosing sources with narrower spectral output width (reducing $\sigma_\lambda$) like using injection laser diode. It is of particular importance for single-mode waveguides and LED system (since an LED has broader output spectrum than a laser diode.)
**Example:** Estimate the rms pulse broadening per kilometer for the fiber with a material dispersion parameter $\left| \frac{\lambda^2}{2} \frac{d^2 n_1}{d\lambda^2} \right|$ of 0.025, and the optical source used is an injection laser diode with a relative spectral width $\sigma_\lambda / \lambda$ of 0.0012 at wavelength of 0.85$\mu$m.

**Solution:**

Material dispersion parameter is given by

$$\lambda = \frac{L}{M} \left| \frac{\lambda \frac{d n_1}{d\lambda} - \frac{1}{c} \frac{d^2 n_1}{d\lambda^2}}{\lambda^2} \right|$$

$$= \frac{1 \times 0.025}{3 \times 10^8 \times 850} = 98.1 \text{ ps/km}$$

The rms spectral width may be obtained from the relative spectral width by:

$$\sigma_\lambda = 0.0012 \lambda = 0.0012 \times 0.85 \times 10^{-6}$$

$$= 1.02 \times 10^{-12} \text{ nm}$$

Therefore, the rms pulse broadening per kilometer due to material dispersion is:

$$\sigma_m = \frac{1.02 \times 10^{-12} \times 10^{-12}}{0.1 \text{ ns/km}}$$
1-2 Waveguide dispersion Waveguide dispersion also occurs because light propagates differently in the core than in the cladding for a particular mode. Waveguide dispersion depends upon the fiber design. The propagation constant which is the function of the ratio of fiber dimension (i.e. core radius) to the wavelength.

In multimode fibers, waveguide dispersion and material dispersion are basically separate properties. Multimode waveguide dispersion is generally small compared to material dispersion. Waveguide dispersion is usually neglected. However, in single mode fibers, material and waveguide dispersion are interrelated.

Dispersion Management: Problem Fiber Dispersion Characteristic

Dispersion Management: Problem Increasing the Bit Rate

Higher Bit Rates experience higher signal degradation due to Chromatic Dispersion:
Dispersion Management: Solution Direct vs. External Modulation

- Laser diode’s bias current is modulated with signal input to produce modulated optical output.
- Approach is straightforward and low cost, but is susceptible to chirp (spectral broadening) thus exposing the signal to higher dispersion.

- The laser diode’s bias current is stable.
- Approach yields low chirp and better dispersion performance, but it is a more expensive approach.

Dispersion Management: Limitation Chromatic Dispersion
CD places a limit on the maximum distance a signal can be transmitted without electrical regeneration:

- For directly modulated (high chirp laser)
  
  \[
  L_D = \frac{1}{B} \frac{1}{|M|} \sigma_\lambda \tag{1}
  \]

  \( M \) dispersion coefficient (ps/km-nm): 17ps/nm*km @1.55μm

  \( \sigma_\lambda \) source line width or optical bandwidth (nm): 0.5nm

  \( B \) bit rate (1/T where T is the bit period): 2.5Gb/s

  \( L_D \sim 47 \) km (*)

- For externally modulated (very low chirp laser \( \sigma_\lambda \sim 1.2B \)).

  \( L_D \sim 1000 \) km @ 2.5Gb/s (*) \[@1.55\mu m \text{ and } 17\text{ps/nm*km} \]

  \( L_D \sim 61 \) km @ 10Gb/s (*) \[@1.55\mu m \text{ and } 17\text{ps/nm*km} \]

**Dispersive properties**

- Anomalous dispersion: \( \beta_2 < 0 \) or \( M > 0 \)
  — short wavelength components (blue) travel faster than long wavelength components (red)

- Normal dispersion: \( \beta_2 > 0 \) or \( M < 0 \)
  — long wavelength components (red) travel faster than short wavelength components (blue)
CD measurements characterize how the velocity of propagation of a light pulse change with wavelength.

CD Coeff. is the slope of the relative GD curve.

Chromatic dispersion coefficient \( D \) (ps/nm-km)

Group delay
Propagation time for a modulated lightwave

\(\tau_g\) (ps)

\(\Delta \lambda\) Bandwidth

\(\Delta T\) Time

Center wavelength
**Dispersion Compensating Fiber:**

By joining fibers with CD of opposite signs and suitable lengths an average dispersion close to zero can be obtained; the compensating fiber can be several kilometers and the reel can be inserted at any point in the link, at the receiver or at the transmitter.

---

**Why Require Dispersion Compensation?**

![Diagram: Relationship of bit rate to dispersion](image)

**CD becomes a serious problem at 10 Gb/s and beyond and leads to a higher bit error rate (BER).**

$$\text{Acceptable amount of CD} \ll \frac{1}{\text{Bitrate}^2}$$
2- Intermodal (Mode or Modal) Dispersion

- Intermodal or modal dispersion results from the propagation delay difference between modes within a multimode fiber. Since modes travel in different directions, some modes travel longer distances. **Modal dispersion** occurs because each mode travels a different distance over the same time span, as shown in the figure. This condition causes the light pulse to spread.

- As the length of the fiber increases, modal dispersion increases.
Modal dispersion is the dominant source of dispersion in multimode fibers. Modal dispersion does not exist in single mode fibers.

In multimode fiber, inter-modal dispersion is the dominant cause of dispersion, but chromatic dispersion can be important at 850 nm.

Intermodal dispersion in a multimode fibers may be reduced by adoption of an optimum refraction index profile which is provided by the near parabolic profile of most graded index fibers.

**Multimode step index fiber**

The delay difference $\delta T_s$ between the meridional ray and the axial ray at the critical angle is

$$\delta T_s = T_{\text{max}} - T_{\text{min}} \approx \frac{L n_1^2}{C n_2} \Delta 1$$

When $\Delta << 1$

The rms pulse broadening $\sigma_s$ resulting from intermodal dispersion mechanism along the multimode step index fiber is

$$\sigma_s \approx \frac{L n_1 \Delta}{2\sqrt{3}C} \approx \frac{L(NA)^2}{4\sqrt{3}C n_1} 6$$

- Requirement for minimal intersymbol interference: $B \Delta t < 1$

Where $B$ = bit rate

- Numerical values for weakly guiding fiber, for which $n_1 \approx n_2 \approx 1.5$:
  1. Step-index multimode ($\Delta \approx 3 \times 10^{-3}$): $BL < 67$ Mb-km/s (MHz-km).
  2. Unclad multimode ($\Delta \approx .33$): $BL < .4$ Mb-km/s (MHz-km).
A 6 km optical link consists of multimode step index fiber with a core refractive index of 1.5 and a relative refractive index difference of 1%. Estimate:

a. The delay difference between the slowest and fastest modes at the fiber output.

b. The rms pulse broadening due to intermodal dispersion on the link.

c. The maximum bit rate that may be obtained without substantial errors on the link assuming only intermodal dispersion.

d. The bandwidth–length product corresponding to (c).

The delay difference is:

\[ \delta T_s = \frac{L \Delta n}{c} = \frac{6 \times 10^3 \times 1.5 \times 0.01}{2.998 \times 10^8} \]

\[ = 300 \text{ ns} \]

The rms pulse broadening due to intermodal dispersion is

\[ \sigma_s = \frac{L \Delta n}{2 \delta T_s c} = \frac{1}{2 \delta T_s} \frac{6 \times 10^3 \times 1.5 \times 0.01}{2.998 \times 10^8} \]

\[ = 86.7 \text{ ns} \]

The maximum bit rate may be estimated in two ways:

1. To get an idea of the maximum bit rate when assuming no pulse overlap.

\[ B_T (\text{max}) = \frac{1}{2 \sigma_s} = \frac{1}{2 \times 86.7 \times 10^{-9}} \]

\[ = 0.7 \text{ Mb/s} \]

2. Alternatively an improved estimate may be obtained using the calculated rms pulse broadening.

\[ B_T (\text{max}) = \frac{0.2}{\sigma_s} = \frac{0.2}{86.7 \times 10^{-9}} \]

\[ = 2.3 \text{ Mb/s} \]
Multimode graded index fiber

- The delay difference $\delta T_g$ is
  \[ \delta T_g \approx \frac{L n_1 \Delta^2}{2C} \approx \frac{L(NA)^4}{8n_1^3C} \]  

- The rms pulse broadening of a near parabolic index profile graded index $\sigma_g$ is reduced compared to the similar broadening for corresponding step index fiber $\sigma_s$ (i.e. the same relative refractive index difference) following
  \[ \sigma_g = \frac{\Delta^2}{D} \sigma_s \]

- Where $D$ is a constant between 4 & 10 depending on the precise evaluation and exact optimum profile chosen.

- The best minimum theoretical rms pulse broadening for GRI fiber with an optimum characteristics refractive index profile for the core $\alpha_{op}$ of
  \[ \alpha_{op} = 2 - \frac{12 \Delta}{5} \]

- The value $\alpha = 2$ produces zero intermodal dispersion in the paraxial approximation of geometrical optics
The total dispersion is given by the following square sum expression:

\[ \sigma_{\text{total}}^2 = \sigma_M^2 + \sigma_{\text{inter}}^2 \]

where \( \sigma_{\text{inter}} \) is the intermodal dispersion of the fiber.

The total propagation delay difference is proportional to \( (\sigma_{\text{total}} \cdot L) \), the fiber bandwidth \( B \) is defined as:

\[ B = \frac{0.2}{\sigma_{\text{total}} \cdot L} \]

This means that the larger the total dispersion and the longer the distance, the lower the transmitted bit rate.

\[ \sigma_{\text{total}} (\text{km}) = \frac{L n_1^2}{2 \pi \Delta^2} = \frac{10 \times 1.5 \times (0.01)^2}{2 \pi \times 2.998 \times 10^8} \approx 1.4 \text{ ps/km} \]

From above the theoretical improvement factor of the graded fiber in relation to intermodal rms pulse broadening is 1000. However, this level of improvement is not usually achieved in practice due to difficulties in controlling the refractive index profile radially over long lengths of fiber.
A multimode-step index fiber has a numerical aperture of 0.3 and a core refractive index of 1.45. The material dispersion parameter for the fiber is 250 ps/nm/km, which makes material dispersion the totally dominating intramodal dispersion mechanism. Estimate (a) the total rms pulse broadening per kilometer when the fiber is used with an LED source of rms spectral width 5 nm and (b) the corresponding bandwidth-length product for the fiber.

(a) The pulse broadening due to material dispersion is:

\[
\sigma_m^2 (1\text{km}) = \frac{\lambda^2}{c} \text{\left(\frac{\partial n}{\partial x}\right)}^2 = \frac{0.85 \times 10^{-11} \times 250 \text{ ps/nm/km}}{1.25 \times 10^8} \times 2.98 \times 10^8
\]

\[
= 12.5 \text{ ns/km}
\]

The total pulse broadening per kilometer due to intermodal dispersion for the step index fiber is:

\[
\sigma_s^2 (1\text{km}) = \frac{L (\Delta n)^2}{\eta c} = \frac{10^8 \times 0.09}{1.25 \times 10^8 \times 1.45 \times 2.98 \times 10^8}
\]

\[
= 29.9 \text{ ns/km}
\]

The total pulse broadening per kilometer (as the waveguide dispersion is neglected)

for the multimode step index is:

\[
\sigma_{opt}^2 (L) = \left(\sigma_m^2 + \sigma_s^2\right)^{1/2} = \left(12.5^2 + 29.9^2\right)^{1/2}
\]

\[
= 34.1 \text{ ns/km}
\]

(b) The bandwidth-length product is:

\[
\frac{B \times L}{10^9} = \frac{a^2}{32.4 \times 10^9}
\]

\[
= 6.2 \text{ MHz/km}
\]
Fiber limitations

<table>
<thead>
<tr>
<th>Type</th>
<th>units/m.km</th>
<th>effecting</th>
<th>typical value</th>
<th>mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>ps/nm</td>
<td>GRI + mono</td>
<td>-20 ps/nm km</td>
<td>Index of refraction of material is a function of wavelength.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mode</td>
<td>-5 ps/nm km</td>
<td></td>
</tr>
<tr>
<td>Waveguide</td>
<td>ps/nm.km</td>
<td>mono mode</td>
<td>-50 ps/nm km</td>
<td>Waveguide propagation constant is a function of wavelength.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-50 ps/nm km</td>
<td></td>
</tr>
<tr>
<td>Multimode</td>
<td>1/2 ps/km</td>
<td>GRI</td>
<td>20 ps/nm km</td>
<td>delta path difference for each mode is translated to</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>70 ps/nm km</td>
<td>delta velocity for each path</td>
</tr>
</tbody>
</table>
LED: $\Delta \lambda / \lambda \approx 0.04$ (Light-Emitting Diode)

FP-LD: $\Delta \lambda / \lambda \approx 0.004$ (Fabry-Perot Laser Diode)

DFB-LD: $\Delta \lambda / \lambda \approx 0.0004$ (Distributed FeedBack Laser Diode)